מקבץ קודים מסכמים – אנליזה נומרית

תיאור המסמך: מקבץ קודים לשיטות נומריות שנלמדו במהלך קורס אנליזה נומרית.

תנאים מקדימים להרצת הקודים:

- התקנת Python מגרסת 3.6 ומעלה.

- התקנת IDE תומך שפת Python , כדוגמת – PyCharm.

- ייבוא סיפריות Scipy ו- Numpy.

- הבנת השיטות שנלמדו בהרצאות.

מחברים: שלי מירון, אור ממן, איוון רובינסון וסתיו לובל.

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**חלק א':**

שיטות למציאת פתרון של משוואה לא לינארית

1. שיטת החצייה

def findRoots(f, range\_start, range\_end, acceptable\_error = 0):  
 """  
 Finds the root of a polynomial based on range.  
 Input  
 f : polynomial/ function  
 range\_start : start range of interval  
 range\_end : end range of interval

acceptable\_error : the acceptable error to stop the loop  
 Output  
 m : the final root of a polynomial based on bisection algo

"""  
 count = 1  
 m = (range\_start + range\_end) / 2.0  
 while (range\_end - range\_start) / 2.0 > acceptable\_error:  
 print(**"Iteration num:"**, count, **", result ="**, m)  
 if f(m) == 0:  
 return m  
 elif f(range\_start) \* f(m) < 0:  
 range\_end = m  
 else:  
 range\_start = m  
 m = (range\_start + range\_end) / 2.0  
 count += 1  
 return m

1. שיטת המיתר

import math  
def findRoots(f, range\_start, range\_end, iterations=10):  
 """  
 Finds the root of a polynomial based on range.  
 Input  
 f : polynomial/ function  
 range\_start : start range of interval  
 range\_end : end range of interval

iterations : number of iteration until   
 Output  
 m : the final root of a polynomial based on secant algo

"""  
 for i in range(iterations):  
 print(**"Iteration num:"**, i, **", result = "**, range\_end)  
 if f(range\_end) - f(range\_start) == 0:  
 return range\_end  
 x\_temp = range\_end - (f(range\_end) \* (range\_end –

range\_start)\* 1.0) / (f(range\_end) -

f(range\_start))  
 range\_start = range\_end  
 range\_end = x\_tem

return range\_end

1. שיטת ניוטון-רפסון

from math import \*  
  
def findRoots(f, derivative, x0=1):

"""  
 Finds the root of a polynomial based on range.  
 Input  
 f : polynomial/ function  
 derivative : A derivative of a polynomial

x0 : a guess of x   
 output  
 x : the final root of a polynomial based on newton-repson

algo  
 """

acceptable\_error = 1e-3  
 x = float(x0)  
 while abs(f(x)) > acceptable\_error:  
 x = x - f(x) / derivative(x)  
 return x

**חלק ב':**

פיתרון נומרי של מערכות משוואות לינאריות

1. שיטת גאוס

def gauss(A):  
 """  
 Solves systems of linear equations using Gauss algo

Input  
 A : the matrix with the solutions of it   
 output  
 x : vector that contains the solutions of the equations   
 """

n = len(A)  
  
 for i in range(0, n):  
 # Search for maximum in this column  
 maxEl = abs(A[i][i])  
 maxRow = i  
 for k in range(i+1, n):  
 if abs(A[k][i]) > maxEl:  
 maxEl = abs(A[k][i])  
 maxRow = k  
  
 # Swap maximum row with current row (column by column)  
 for k in range(i, n+1):  
 tmp = A[maxRow][k]  
 A[maxRow][k] = A[i][k]  
 A[i][k] = tmp  
  
 # Make all rows below this one 0 in current column  
 for k in range(i+1, n):  
 c = -A[k][i]/A[i][i]  
 for j in range(i, n+1):  
 if i == j:  
 A[k][j] = 0  
 else:  
 A[k][j] += c \* A[i][j]  
  
 # Solve equation Ax=b for an upper triangular matrix A  
 x = [0 for i in range(n)]  
 for i in range(n-1, -1, -1):  
 # Round - approximation  
 x[i] = round(A[i][n]/A[i][i],3)  
 for k in range(i-1, -1, -1):  
 A[k][n] -= A[k][i] \* x[i]  
 return x

**חלק ג':**

שיטות איטרטיביות לפתרון של מערכות לינאריות

1. שיטת יעקובי

import numpy as np  
  
def jacobi(A, b, ITERATION\_LIMIT = 1000):  
 """  
 Solves systems of linear equations using Jacobi algo

Input  
 A : matrix of linear equations

b : solutions of linear equations

ITERATION\_LIMIT : max iteration till stop  
 output  
 x : vector that contains the solutions of the equations   
 """

x = np.zeros\_like(b)  
 for it\_count in range(ITERATION\_LIMIT):  
 x\_new = np.zeros\_like(x)  
  
 for i in range(A.shape[0]):  
 s1 = np.dot(A[i, :i], x[:i])  
 s2 = np.dot(A[i, i + 1:], x[i + 1:])  
 x\_new[i] = (b[i] - s1 - s2) / A[i, i]  
  
 if np.allclose(x, x\_new, atol=1e-10, rtol=0.):  
 break  
  
 x = x\_new  
 # error = np.dot(A, x) - b  
 return x

1. שיטת גאוס-זיידל

import numpy as np  
from scipy.linalg import solve  
  
def gaussSeidel(A, b, x, n):  
 """  
 Solves systems of linear equations using Gauss-zidel algo

Input  
 A : matrix of linear equations

b : solutions of linear equations

x : vector that contains the solutions of

the equations

n : number of iteration

Output  
 x : vector that contains the solutions of the equations   
 """

L = np.tril(A)  
 U = A - L  
 for i in range(n):  
 x = np.dot(np.linalg.inv(L), b - np.dot(U, x))  
 print (**'**\n**'**,**'Iter '**, i, **':'**)  
 print(x)  
 return x

**חלק ד':**

שיטות אינטרפולציה

1. שיטת האינטרפולציה לפי לאגרנז'

import scipy.interpolate as interpol

"""  
 We calculate lagrange interpolation by sending 3 points and

receiving function back  
 Input  
 xp: input x's in a list of size n  
 yp: input y's in a list of size n  
 Output  
 f : the polynomial of degree n-1  
 """  
  
f = interpol.lagrange(xp, yp)  
print(f)  
print(**'f({0}) = {1}'**.format(x , f(x)))

1. שיטת אינטרפולציה לפי נוויל

def neville(datax, datay, x):  
 """  
 Finds an interpolated value using Neville's algorithm.  
 Input  
 datax: input x's in a list of size n  
 datay: input y's in a list of size n  
 x: the x value used for interpolation  
 Output  
 p[0]: the polynomial of degree n  
 """  
 n = len(datax)  
 p = n\*[0]  
 for k in range(n):  
 for i in range(n-k):  
 if k == 0:  
 p[i] = datay[i]  
 else:  
 p[i] = ((x-datax[i+k])\*p[i]+ \  
 (datax[i]-x)\*p[i+1])/ \  
 (datax[i]-datax[i+k])  
 print(**'P{0}{1} = {2}'**.format(i, k, p[i]))  
 return (**'Result => P{0}{1}({3}) = {2}'**.format(i, k, p[0],x))

1. שיטת ספליין-קובי

**חלק ה':**

שיטות אינטגרציה וגזירה נומריות

1. שיטת הטרפז

import numpy as np  
  
def calculate\_area(f, a, b, n):

"""  
 Calculate the integral of a f(x) based on the trappezodial rule.  
 Input  
 f : the polynomial/ function

a : the start range of an integral

b : the end range of an integral  
 n : number of interval  
 Output  
 np.trapz(f(x), x): the integral of f(x)  
 """  
 x = np.linspace(a, b, n + 1)

print(**"number of intervals: "**, n+1)  
 return np.trapz(f(x), x)

1. שיטת סימפסון

from scipy import integrate  
  
def simpson(y, x):

"""  
 Calculate the integral of a f(x) based on the simpson rule.  
 Input  
 y : y's points – y range of a polynomial

x : x's points – x range of a polynomial

Output

Integral of a polynomial based on particular points

"""

return integrate.simps(y, x)  
  
print(**"Integral:"**, simpson(y, x))

1. שיטת רומברג

from scipy import integrate  
import numpy as np  
  
def romberg(f, a, b):  
 """  
 Calculate the integral of a f(x) based on the romberg rule.  
 Input  
 f : polynomial/ function

a : x range of integral

b : y range of integral

Output

Integral of a polynomial based on range

"""

return integrate.romberg(f, a, b, show=True)  
print(**"Integral: "**, romberg(f, a, b))

1. תרבועי גאוס

from scipy import integrate  
  
"""  
Returns:   
val : float  
Gaussian quadrature approximation (within tolerance) to integral.  
  
err : float  
Difference between last two estimates of the integral.

"""

result = integrate.quadrature(f, a, b)  
print(result)

**נספחי קוד**

1. חישוב מטריצה הופכית

def invert\_matrix(A):

return linalg.inv(A)

1. חישוב נורמה של מטריצה

def Norma(A):

sum = 0  
 temp\_sum = 0  
 for i in range (len(A)):  
 if temp\_sum >= sum:  
 sum = temp\_sum  
 temp\_sum=0  
 for j in range (len(A)):  
 temp\_sum += abs(A[i][j])  
 return sum

1. חישוב Cond

def cond(A):

return Norma(A)\* Norma(invert\_matrix(A))

1. חישוב LU

import pprint  
import scipy

P, L, U = scipy.linalg.lu(A)